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Automated Approach to Design of Solid Rockets

Richard H. Sforzini*
Auburn University, Auburn, Ala.

An improved pattern-search technique is utilized in a computer program which minimizes the sum of the squares of the differences, at various times, between a desired thrust-time trace and that calculated with a mathematical performance model of a solid-propellant rocket motor (SRM). Up to 14 design parameters may be varied during the search. These parameters fix the design of the SRM which may consist of various common grain configurations. The large number of designs that must be evaluated during the search for the "best match" of calculated to desired performance necessitates the use of a relatively simple model of the SRM's performance. The one selected provides the required data for a single design on the IBM 3031 computer in about 10 s of operating time. An important feature of the model is an approximate evaluation of the effects of grain deformation on burning surface geometry. The performance model is shown to give good quality predictions for 5 existing SRM's of very different sizes and configurations. The overall program is demonstrated by matching the thrust-time trace obtained from the static test of the first Space Shuttle SRM starting with input values of 10 variables which are, in general, 10% different from the as-built SRM. An excellent match is obtained; final design parameters are for the most part within a few percent of the as-built values.

Nomenclature

a_0	= propellant burning-rate coefficient at zero crossflow velocity (in./s-psi^n)
D_p, D^*	= port and throat diameter, respectively (in.)
f	= fillet radius of star grain (in.)
F	= thrust (lbf)
G	= mass flow rate per unit area (slug/s-in.^{2})
L	= flow length in burning rate law, Eq. (1) (in.)
L_c, L_n, L_s, L_t	= defining lengths used as variable design parameters (see Fig. 2) (in.)
n	= burning-rate exponent
q_i	= weighting factor in objective function, Eq. (2)
P	= pressure (psia)
P_j	= penalty function of jth constraint
r	= burning rate (in./s)
t	= time (s)
W_m	= motor mass (lbm)
X_t	= difference in web thickness at the ends of L_t (see Fig. 2) (in.)
y	= distance burned normal to the propellant surface (in.)
Z	= initial difference between web thickness at the head and nozzle ends of the controlling grain length (see Fig. 2) (in.)
θ_g	= defining angle used as variable design parameter (see Fig. 2) (deg)
α_n	= nozzle exit half angle (deg)
δ	= indicates percentage change in optimization variables (5% change for each unit of δ)
Δ	= change or difference in a quantity
T_s, T_t, T_w	= length-averaged web thickness for a standard-star, slotted-tube, or wagon-wheel grain configuration, respectively (in.)
Φ	= objective function (lbf^2)

Subscripts

e	= exit plane
i,j	= running indices for time points and constraints, respectively. i also stands for inside of grain
n	= nozzle end of grain at position of maximum flow Mach number
o	= outside of grain
max	= maximum value

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*Professor, Aerospace Engineering Department. Associate Fellow, AIAA.

Introduction

THE engineering design process is a search for a set of parameters that fix the configuration and other properties of the end item in a way that will best meet certain performance criteria. In general, the criteria involve stated goals for the performance to be achieved and practical limits on the design parameters. Mathematically, the problem may be described as one of finding the optimum value of a function of a number of variables subject to a set of inequality constraints on a number of these variables. Although the problem is simply stated, application of the formal mathematical techniques suggested by the description to thrust-producing devices appears a rarity.

Because the solid-propellant rocket motor (SRM) is a relatively simple device with a reasonably small number of parameters to be fixed, its design process is especially amenable to a formal mathematical approach. However, limited treatment of this subject appears in the literature. Woltosz[1] demonstrates the application of a mathematical optimization technique to the selection of SRM design parameters that meet certain design goals. In particular, for an SRM of the Space Shuttle (SS) class, he shows how a set of 5 critical design dimensions are determined which, for a specified minimum ideal vehicle velocity, maximize the total impulse-to-motor weight ratio (or alternatively minimize the motor weight) without exceeding a specified maximum allowable web fraction. For this purpose, he develops a computer program that uses the direct pattern-search optimization technique of Hooke and Jeeves[2] as adapted into a computer program by Whitney.[3] The large number of complete ballistic analyses necessary dictates a practical requirement for a relatively simple mathematical model of the design and operating characteristics for the optimum SRM. To this end, Woltosz adapts the simplified design analysis computer program of the present author.[4,5]

The pattern-search technique has also been employed in SRM igniter design by Foster and Sforzini[6,7] to minimize the difference between certain desired main-motor performance characteristics and those calculated using a simplified mathematical model[5,8] of the ignition transient. For example, the set of 5 igniter design characteristics is selected which minimizes the summation of the absolute values of the difference between the desired and computed SRM values during ignition.

In the present paper, a practical computer program similar to the igniter optimization is demonstrated which will match *overall* main-motor desired performance, i.e., a specified thrust-time trace, to calculated performance by optimum determination of main-motor design parameters. This is accomplished by systematic adjustments of the design parameters (the pattern search) until the minimum difference between the specified and theoretical (predicted) traces is obtained. The computer program of Woltosz is modified to include the new optimization criterion while retaining the two separate criteria of the original program as options. In addition, extensive revisions and additions are made, 1) to provide a better performance model so that the search will accurately converge, 2) to improve the pattern-search technique for the purpose of reducing computer time, 3) to specify more adequately the design constraints that assure a set of final design parameters that describe an SRM of practical design, and 4) to include additional design variables. These modifications are discussed next. Finally, a demonstration of the program is presented with 10 design parameters as variables.

The Performance Model

A good mathematical model of the performance of single SRM's is essential to the ordinary design process. For the automated design approach to be reliable, accuracy of the model is especially important; otherwise, the search in an effort to eliminate all differences between the calculated and required values will incorrectly adjust the parameters. The accuracy requirements seem to call for a very sophisticated model which conflicts with the requirements for a model permitting very rapid calculations, since the search typically requires several hundred complete ballistic evaluations.

A solution to the accuracy-with-speed requirement has been found which will give an adequate performance evaluation for a single design in about 10 s of operating time on the IBM 3031. This is achieved by several modifications of the simplified design analysis program described in detail in Refs. 4 and 5 and summarized in Ref. 9. The principal modifications are 1) the evaluation of the effects of propellant deformation on the burning surface and hence on the internal ballistics and 2) the inclusion of a special empirical equation which gives the erosive burning rate of the propellant.

Smith[10] shows how propellant deformation can affect the burning rate of the propellant by distortion of the burning surface and account for a significant portion of the "scale factor" between large- and small-motor burning rates. The important element is the tangential strain at the burning surface. Smith's analysis has previously been coupled with the simplified performance analysis and in general yields significant improvements in performance predictability.[7,11,12] The deformation analysis is strictly applicable only to circular-perforated (c.p.) grains or grain segments. However, at least for certain star-grain configurations, the deformation effect is probably small.[12]

The major key to the relative simplicity of the basic performance model used is the assumption that the burning surface regresses linearly along the grain length consistent with the differences in the regressions at the head and aft ends, for which burning rates are separately calculated with erosive burning included at the aft end.[9] This approach eliminates the need for the time-consuming iteration needed to determine the mass flow rate and combustion-chamber pressure. Although the validity of the assumption may be questioned, especially as to its influence on tailoff predictions, the final results tend to show that a good assessment of the erosive burning at the aft end of the grain coupled with the simplified model including the deformation effect provides an adequate degree of overall predictability. The accurate determination of the lateral burning rate r_n at the aft end of the propellant port is thus fundamental to the analysis.

The empirical equation used to determine r_n in terms of the zero crossflow-velocity burning rate r_0, SRM pressures, geometry, and flow parameters is

$$1/r_n = 1/r_0 - 265.3(G/L)^{0.5}(D_p/L)^{0.8} \times [1 - 3755 r_n^{0.2}(D^*/L)^{0.7}/P_n] \tag{1}$$

Details of the development of this relationship are given in Refs. 7 and 11-13. That the overall performance model gives reasonably accurate results is demonstrated by using it to

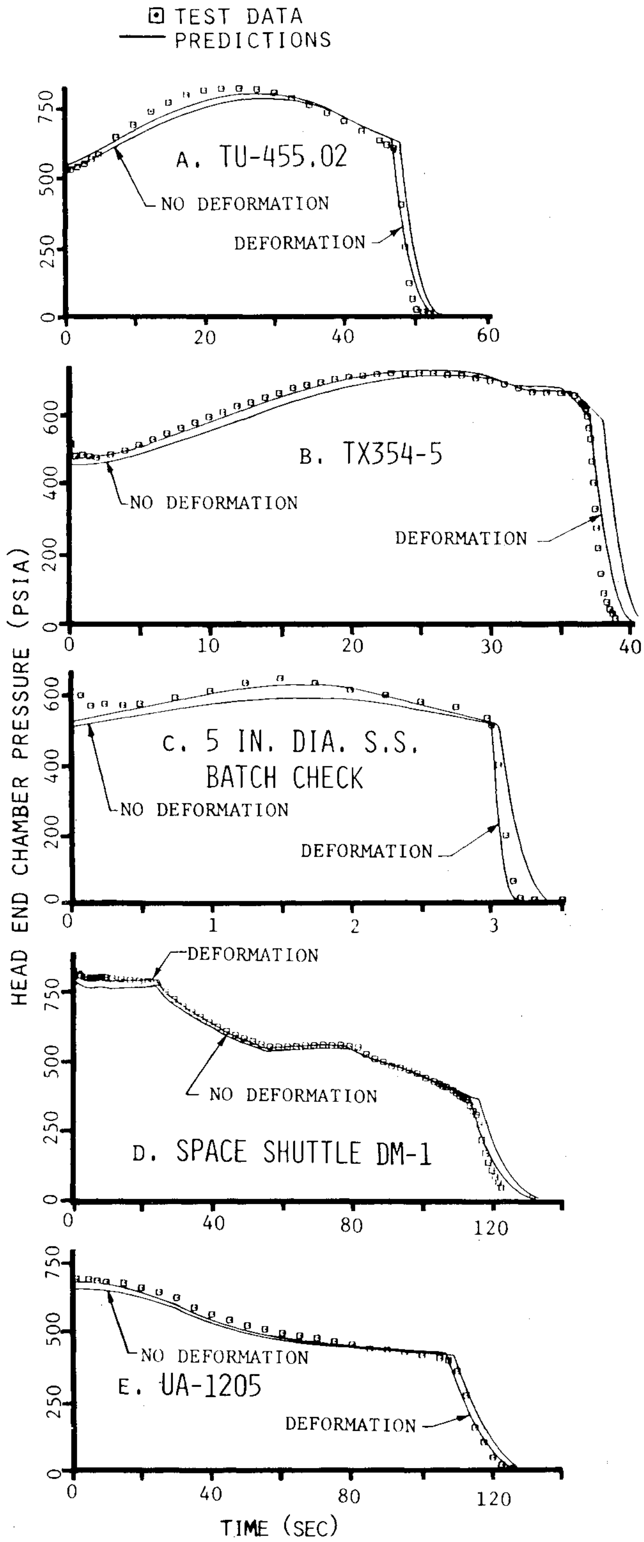

Fig. 1 Comparison of test data on SRM's with theoretical results.

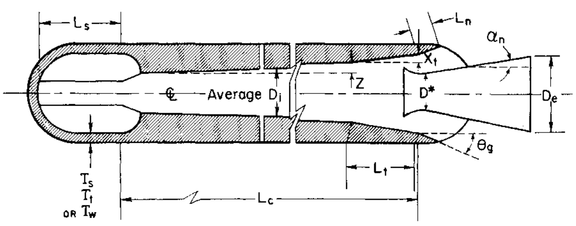

Fig. 2 Variable design parameters fixed by SRMDOP.

predict the chamber pressure vs time of the following SRM's: 1) the first Space Shuttle Development Motor (DM-1), 2) the Space Shuttle 5-in. diameter ballistic test motor, 3) the TU-455.02, 4) the Castor TX354-5, and 5) the Titan IIIC/D (UA 1205). These SRM's all use high-solid-level propellant with ammonium perchlorate oxidizer and polybutadiene-acrylic acid-acrylonitrile terpolymer binder except for castor which uses a carboxzl-terminated butadiene binder. The propellants all contain from 16 to 20% aluminum.

Partial results for the 5 SRM's are presented in the pressure vs time traces given in Fig. 1. Where thrust data were available for comparison, the thrust-time traces were of the same quality as the pressure-time traces. The latter are shown rather than the former in order to demonstrate that model deficiences are not masked in the thrust correction factor applied in the analysis to the thrust calculations based on ideal nozzle flow theory. The ideal thrust itself is based on nozzle-end stagnation pressure with due regard to mass-addition effects. The nozzle correction factors (constant in the program) for the SRM's of Fig. 1 are determined by comparison of the thrust-time and pressure-time traces from the test data. For design of a new SRM, the thrust correction factor from existing motors of similar design would be used or the value deduced from analytical programs that account for nonideal factors such as multidimensional and multiphase flow. In Fig. 1, results are given with and without grain-deformation effects and compared with actual performance data. The ballistic performance evaluation show that better results are obtained when the propellant deformation is taken into account.

The Optimization Program

The new optimization computer program is entitled Solid Rocket Motor Design and Optimization Program (SRMDOP). The complete computer program is described in detail in Ref. 13. The present paper summarizes the program and identifies the major analytical modifications and additions to the original program of Woltosz.[1] The inclusion of the special burning-rate law has already been discussed.

The SRMDOP systematically adjusts by the pattern-search method the variables which fix the SRM design such that it meets the optimization criterion specified. Any number of 15 geometric variables which are defined by Fig. 2 may be so treated, but only 1 of the 3 variables T_s, T_t and T_w may be used in a single problem as these are the web thicknesses corresponding to 3 different types of grains—standard-star, slotted-tube, and wagon-wheel configurations, respectively.[4,5] The zero cross-velocity burning-rate coefficient a_0 may be used as an additional variable, thus permitting the use of up to 14 variables in fixing a design.

The version of the simplified design analysis program[4,5] that forms the basis for the performance model as adapted by Woltosz was quite restricted in its treatment of fore and aft grain geometry where star grains are involved. A vital improvement is made in the new program by restoring the option of representing special propellant burning geometry through the input of tabular values of burning surface vs distance burned. This permits the program to treat, for example, a star grain intersecting a curved closure, a transition section between star and circular-perforated (c.p.) grain sections, or the effect of partial inhibitors on the grain ends as in the Space Shuttle SRM's. However, in the use of this option, it may be necessary to approach the design in stages since the surface vs distance burned associated with some of these special sections cannot be determined until the final design parameters are fixed. In the first stage, the special surface vs distance burned trace would be separately determined based on the starting (input) configuration. Once the program has determined the optimum set of variables based on this input, a new burning surface can be determined and used as input to redetermine the optimum variables. A few iterations of this type should suffice to fix an accurate solution. Of course, if the variables used do not affect the geometry of the special sections, as is often the case, iteration is unnecessary.

Woltosz uses relatively few constraints on input variables. However, he constrains D_e to be greater than 2 D^* and less than the case diameter, α_n to be less than and greater than specified values, and the web fraction (controlled by D_i) to be within specified limits. Also, a fixed lower limit (0.5 in.) is set for f, the fillet radius of a star grain. Other lengths and angles are required to be ≥ 0. Such constraints are required to prevent the program from selecting impractical or absurd values for the variable design parameters. In SRMDOP, the upper limit on D_e is arbitrary. The range of initial throat diameter D^* is set by upper and lower limits on the throat-to-port area ratio. The lower limit on f is an input. Upper limits are also specified for L_t, X_t, Z, θ_g, L_c, L_n, L_s, f, T_s, and T_w. The first seven of these latter upper limits are established within the program. For example, L_t must meet the internal limit of being less than one-half the estimated total grain length L, and X_t must be less than 10% of L_t.

Two new design optimization variables have been added to the program. These are T_w, the web thickness of a wagon-wheel grain or grain segment, and a_0, the propellant zero-crossflow-velocity burning-rate coefficient. Both of these new variables are constrained by specified arbitrary upper and lower limits.

As in the original program, constraints are enforced by the use of penalty functions which, by adding or subtracting a large number to the objective function (quantity to be optimized), force the search away from conditions that cause the constraints to be violated. The value of the penalty is assigned in proportion to the square of the amount by which the constraint is violated. To avoid difficulties with possible mathematical discrepancies that would be encountered when a constraint is violated, in addition to assigning a penalty function, the value of the variable is set equal to the constraining value whenever a constraint is violated.

As previously indicated, there are three options, any one of which may be selected as a single optimization criterion. The two criteria used by Woltosz are retained: 1) maximize the ratio of total impulse to motor weight and 2) minimize motor weight. Each of these criteria is subject to the side condition, enforced by the use of a penalty function, that a specified minimum ideal vehicle velocity be obtained. Here the ideal velocity is that obtainable at burnout with no payload, no drag, and no gravity. Such optimization is of some value in design when little or nothing is known of the mission for the rocket. The program could easily be modified to include payload, drag, and gravity considerations in the constraining equation.

In the new program, a third option has been added to force the search to select a set of design parameters that will give the predicted thrust-time trace that most nearly matches a desired thrust-time trace. The desired trace is specified by input of a discrete number of thrust values and their corresponding time points. The program calculates the thrust values at the same time points for each set of design parameters assigned during the search. The objective function Φ is then given by the following equation:

$$\Phi = \sum_i q_i (\Delta F_i)^2 + \sum_j P_j \tag{2}$$

Table 1 Comparison of starting and final values with the values representing the as-built configuration of the DM-1 for the demonstration program

(1) Variable (see Fig. 2)	(2) Starting value	(3) Final value	(4) As-built representation	% Difference (3) vs (4)	% Difference Alternative design[a] vs (4)
D^*, in.	48.988	54.052	54.420	−0.7	−14.4
L_t, in.	219.82	258.28	244.25	+5.7	+12.6
Z, in.	2.90	2.41	3.27	−26.3	−8.5
D_i, in.	59.27	60.57	59.87	+1.2	+3.9
a_0, in./s-psia^n	0.0449	0.0411	0.0409	+0.5	−10.0
θ_g, deg	8.10	8.10	9.00	−10.0	−10.0
L_c, in.	1027.17	1130.94	1141.13	−0.8	−0.8
L_n, in.	50.94	55.62	56.60	−1.7	−9.0
L_s, in.	159.55	171.39	177.28	−3.3	−9.1
T_t, in.	7.92	9.03	8.80	+2.5	−0.6
$\Phi \times 10^{-3}$, lbf^2	133.19	0.389	0.710	−45.2	−73.1
$W_m \times 10^{-6}$, lbm	0.998	1.087	1.106	−1.7	b
$P_{\max}$, psia	1154.0	844.0	835.0	+1.1	b
Comp. time, min	...	33.15	...		...

[a]Starting values for the alternative design are the same as (2) except $a_0 = 0.03678$. [b]Final ballistic and motor weight data were not printed for the alternative design.

Here ΔF_i is the difference between the desired and predicted thrust at the ith time point and q_i is an arbitrary weighting factor to be assigned to each thrust-time point. The penalty function for the jth constraint is P_j. It is zero if the constraint is not violated otherwise it is a large positive number multiplied by the square of the amount by which the constraint is violated. The minimization of Φ thus leads to the condition where all constraints are satisfied ($\Sigma_j P_j = 0$) and $\Sigma_i q_i (\Delta F_i)^2$ is a minimum. This in turn yields the "best match" between predicted and desired performance.

The factor q_i is used to assign relative importance to obtaining design goals for various portions of the trace. Also it can be used to prevent the search from being misled because of inaccuracy in the modeling of the ballistics. For example, in the demonstration program that follows, relatively low values are assigned to q_i for the last two data points. This is because the program presently does not model nozzle-flow separation such as occurs at very low chamber pressure under the conditions prescribed. This defect can be easily corrected but, for the present, its impact on the results is minimized by the expedient described.

Two major modifications of the original pattern-search technique[3] are incorporated in the new program. With the original approach, excessive time is consumed in the program in both exploratory and pattern moves[1,2] involving only variables to which the performance (thrust vs time) is only slightly sensitive. The search tries to attain the minimum Φ by repeated adjustment of these variables at the same step size δ while holding the other variables constant, because the effects of the latter variables are so sensitive that at the larger step sizes they produce large values of Φ. The solution is to disregard the changes produced by the variables which affect performance only slightly and to proceed directly to the reduction of δ. Of course, at reduced step size the sensitivity of all variables becomes important. The overall problem is solved in the program by rejecting moves for which

$$\Delta\Phi < 0.05\Phi\delta^{1.5} \tag{3}$$

Here $\Delta\Phi$ is the change in Φ during a move and $\delta = 1$ corresponds to 5% of the variable for which a change of less than 5% in Φ would be rejected. The exponent of δ was fixed by trial and error to provide an efficient search that permits the variables with small effects to be appropriately adjusted as the step size is reduced.

The second major change made in the pattern search to improve its efficiency is the elimination of exploratory moves about the end values of an unsuccessful pattern move. In general, in this investigation such exploratory moves did not prove successful. The procedure now is to return to the previous basepoint and continue the search with exploratory moves at reduced step size. Each time a reduction is made, the step size is reduced by a factor of 2 until the minimum (specified) step size is reached.

The success of the results is demonstrated by comparing Woltosz's results to those obtained with the present program. Woltosz, on a CDC 6400 computer, obtained an optimum design for maximum total impulse/motor weight with 5 variables in somewhat over 1 h of computer operating time.[1] In the following demonstration with the present program, more or less optimum thrust-time traces are obtained with 10 variables on the IBM 3031 in less than 34 min of computer operating time.

As a test of the program's capabilities, an attempt is made to match the thrust vs time trace obtained from the static test of the first Space Shuttle SRM (DM-1) starting with input values, all except two of which are 10% below the values used to represent the DM-1 configuration in the simplified design analysis program. For a reason to be given later, a_0 is set to a value that is 10% above and D_i is set to a value 1% below the corresponding values for DM-1. The invariant design parameters, such as propellant density and characteristic velocity, case outside diameter, nozzle exit half angle, and nozzle erosion rate, are set at the values used to represent the as-built configuration. Ranges of variables are set by assignment of constraints consistent with good design practice. For example, the upper and lower limits on the throat-to-port area ratio are set (for this demonstration only) at 0.84 and 0.20, respectively, with a view toward avoiding excessive erosive burning and low volumetric loading density.

Results for the example problem are best summarized by a comparison of the final values of the variable design parameters with those of the as-built representation of DM-1 configuration. This is done in Table 1 where the starting values and percent differences between final and as-built values are also given. It is seen that the final values are, for the most part, within a few percent of the as-built values. One exception is Z where a substantial difference exists because of the inability of the simplified design analysis to fully model the taper of the two central and identical c.p. grain segments in the DM-1. A similar difficulty arises with θ_g because the actual aft-end configuration of the Space Shuttle SRM can only be approximated with the variable. This is the reason reference is made to the as-built *representation*.

Table 1 also gives the objective function (Φ) for the three configurations evaluated by SRMDOP including the as-built

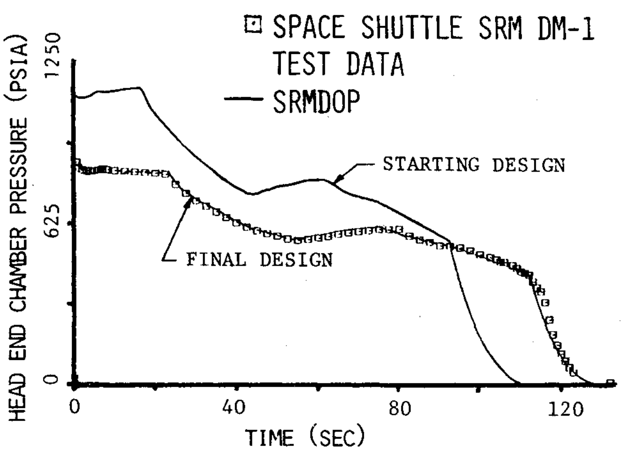

Fig. 3 Head-end chamber pressure vs time from DM-1 test data and from SRMDOP.

representation. It is seen that the final values obtained from the search yield a slightly lower Φ (389) than that of the as-built representation. This is to be expected since the final values are obtained as a result of a search wherein the 10 optimization variables are free to adjust their values in an attempt to compensate for differences between desired and calculated thrust that result from deficiencies in the modeling. Nevertheless, the modeling is good enough and the objective functions are low enough in both cases to provide assurance that, if two SRM's were built to the two different sets of specifications, the performance of each would be very close to that desired.

An additional summary of the program results is a plot in Fig. 3 of head-end chamber pressure vs time for the initial and final configurations. Figure 3 shows that the actual chamber pressure of the DM-1 is well matched by the final design furnished by the SRMDOP, and the thrust, as evidenced by the low Φ value, is also well matched.

The results of the demonstration show SRMDOP offers an approach to preliminary design of SRM's that has great potential for simplifying and shortening the design process. However, the designer must first develop an understanding of the program and how it responds. For example, when tabular values are required to specify portions of the burning surface, special consideration needs to be given to dividing the burning surface properly into those sections to be represented by equations and those to be represented by tabular values.

One difficulty occurs in the first attempt to conduct this demonstration and indicates at first a weakness but later a strength of the program. This difficulty is encountered when the value of all 10 variable design parameters are input 10% below the as-built values. The low burning rate combines with the low value of D^* in such a way that, after a few moves in the search, a rather low value of the objective function is obtained with a low burning rate compensating for a low D^*. The program then proceeds to optimize the other variables around these values of D^* and a_0 with a final Φ value of 465. This is evidence of a tendency in the program to seek out local minima rather than the true or global minimum. The particular minimum towards which the program converges depends on the starting values of the variable design parameters. For the demonstration, it is necessary to show that the program can converge toward the as-built configuration from off-design values. Good results are obtained by starting with an a_0 that is 10% higher than the as-built values, rather than 10% lower, and a D_i that is only 1% lower than the as-built values. As previously shown, this gives a Φ of 389 which is lower than that for the as-built representation. It is also lower than that which results when all 10 starting values are 10% lower than the as-built design.

A lower Φ does not necessarily indicate better convergence towards the as-built design. Indeed, when the program is started with all values 10% below the as-built values, except for D_i which is started at 1% less than its as-built value, an Φ of 191 is obtained. The corresponding percentage deviation of final values of the ten design parameters from the as-built values are given in the last column of Table 1 under the heading of "Alternative design." It is seen that the deviations are substantial, although the lower value of the objective function shows that a somewhat better trace match is obtained.

The strength of SRMDOP brought out by the difficulty encountered in the demonstration is that, by using a number of sets of starting variables, a number of designs can be found that will satisfy to varying degrees the performance requirements specified within the limits imposed by the goodness of the mathematical model of the internal ballistics. The designer is then free to make a final selection based upon consideration of other characteristics as well as the objective function. For example, he might decide that the smaller throat of the "alternative design" might be desirable and the lower Φ of this design would provide some assurance that a better trace shape would be obtained than with the final design parameters of the primary demonstration.

If the lack of uniqueness of the solution is a concern, additional constraints could be easily incorporated which would limit the range of possible solutions. For example, a constraint on maximum pressure could be provided simply by adding another P_j to Eq. (2). Since the program can calculate both propellant and inert weight, constraints on total motor weight, impulse-to-motor weight ratio, or both can be similarly enforced.

Simple single or double variable optimizations can be conducted with SRMDOP for the purpose of determining how to alter an existing design to achieve more desirable thrust-time trace characteristics. Also, the program might be used to deduce from test data the apparent burning-rate coefficient a_0 based on either $r = a_0 p^n$ with no erosive burning or the burning-rate relation given by Eq. (1). Alternative burning-rate laws could also be investigated with appropriate program modifications.

Of course, improvements of the program are possible. An obvious one that has been discussed earlier is to incorporate in SRMDOP a model of flow separation during tailoff. Another improvement that would add to the flexibility of the program would be to include the outside diameter of the propellant (D_o for c.p. grains) as a design variable. Presently, the designer can investigate the potential of various grain diameters through several runs with various fixed D_o and corresponding diameters for star-grain segments.

Concluding Remarks

The program described should provide the designer with a powerful approach to preliminary design. The simplified performance model appears to give the accuracy in predictions required for reliable results and permits solutions within reasonable computer operating times. The comparisons of predictions with experimental data tend to confirm both the hypothesis that grain deformation affects ballistic performance and the way the deformation is taken into account in the program.

Acknowledgments

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References

[1] Woltosz, W.S., "The Application of Numerical Optimization Techniques to Solid-Propellant Rocket Motor Design," M.S. Thesis, Auburn University, Auburn, Ala., March 1977.

[2]Hooke, R. and Jeeves, T.A., "Direct Search Solution of Numerical and Statistical Problems," *Journal of the Association of Computing Machinery*, Vol. 8, No. 2, Feb. 1961, pp. 212-229.

[3]Whitney, D.W., "Pattern Search," Civil and Mechanical Engineering Joint Computer Facility, Massachusetts Institute of Technology, Cambridge, Mass., April 1974.

[4]Sforzini, R.H., "Design and Performance Analysis of Solid-Propellant Rocket Motors Using a Simplified Computer Program," Final Report, NASA Contractor Report CR-129025, Auburn University, Oct. 1972.

[5]Sforzini, R.H., "Extension of a Simplified Computer Program for Analysis of Solid-Propellant Rocket Motors," Final Report, NASA Contractor Report No. CR-129024, Auburn University, April 1973.

[6]Foster, W.A. Jr. and Sforzini, R.H., "Optimization of Solid Rocket Motor Igniter Performance Requirements," Paper No. 78-1016, 14th AIAA/SAE Joint Propulsion Conference, Las Vegas, Nev., July 1978.

[7]Sforzini, R.H., Foster, W.A. Jr., and Murph, J.E., "Solid-Propellant Rocket Motor Internal Ballistics Performance Variation Analysis (Phase Four)," Final Report prepared for NASA, Auburn University, Jan. 1979.

[8]Sforzini, R.H. and Fellows, H.L. Jr., "Prediction of Ignition Transients in Solid-Propellant Rocket Motors," *Journal of Spacecraft and Rockets*, Vol. 7, May 1970, pp. 636-638.

[9]Sforzini, R.H. and Foster, W.A. Jr., "Monte Carlo Investigation of Thrust Imbalance of Solid Rocket Motor Pairs," *Journal of Spacecraft and Rockets*, Vol. 13, April 1976, pp. 198-202.

[10]Smith, D.F., "Solid Propellant General Deformations and Their Effects on Motor Performance," Master of Science Thesis, Auburn University, Dec. 1976.

[11]Sforzini, R.H. and Foster, W.A. Jr., "Solid-Propellant Rocket Motor Internal Ballistic Performance Variation Analysis (Phase Two)," Final Report, NASA Contractor Report CR-150324, Auburn University, Sept. 1976.

[12]Sforzini, R.H., Foster, W.A. Jr., Murph, J.E., and Adams, G.W. Jr., "Solid-Propellant Rocket Motor Internal Ballistics Performance Variation Analysis (Phase Three)," Final Report, NASA Contractor Report CR-150577, Auburn University, Nov. 1977.

[13]Sforzini, R.H. and Murph, J.E., "Solid-Propellant Rocket Motor Internal Ballistics Performance Variation Analysis (Phase Five)," Final Report prepared for NASA, Auburn University, Jan. 1980.